Lightweight Max Weight Scheduling Algorithms for Wireless Networks

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***Abstract*—We propose a class of binary queue length information based max-weight scheduling algorithms for wireless networks. In these algorithms, the scheduler, in addition to channel states, only needs to know when a link's queue length crosses a prescribed threshold. We show that these algorithms are throughput optimal. Further, we incorporate time-since-last-service (TSLS) information to improve delay and service regularity of the scheduling algorithms while ensuring throughput optimality. We also perform simulation to illustrate throughput, delay and service regularity performance of the proposed algorithms.**

I. Introduction

In the recent past, there has been increasing deployment of Internet of Things (IoT), bringing in huge number of traffic generating devices meant to communicate over a shared wireless medium. Enormous number of IoT devices and broadcast nature of wireless channel require careful management of interference among simultaneous transmissions, an operation commonly referred to as *scheduling*. The scheduling algorithms, besides stability of networks, should also address nodes' quality-of-service (QoS) needs, e.g., delay, inter-service times etc. Further, IoT devices are typically spectrum and energy limited. Hence, the generating good schedules should not require excessive communication.

There has been extensive work on queue-length based wireless scheduling protocols which are shown to optimize the throughput performance of wireless networks. Landmark work of Tassiulas and Ephremides [1] characterized the capacity region of the networks and also showed that a Queue length based max-weight scheduling algorithm, referred to as Max-Q here, renders node queues stable for any arrival rates for which any other scheduling strategy can keep them stable. However, Max-Q requires each device's medium access control (MAC) module to know its queue length (amount of backlogged data) and also to communicate this to a scheduler in each slot. While the former violates the basic proposition of layered network architecture, the latter entails substantial bandwidth and energy costs. In heterogeneous networks where IoT nodes with sparse traffic share wireless medium with other nodes generating heavy traffic, Max-Q can also lead to poor delay and service regularity performance for the former ones because it favours links with larger queue lengths.

We propose a throughput optimal max-weight scheduling algorithm, Max-W, in which the MAC only needs to inform the scheduler whenever the data queue crosses a threshold (possibly 0). A links's weight in any slot, which is a function of the time for which the queue length has been above this threshold (e.g., has been nonempty), is then known to the scheduler. So, these algorithms eliminate cross layer iteration and also economize on the communication overhead. Expectedly, the delay performance of Max-W is inferior to that of Max-Q. We next propose a variant, Max-WT, where link weights are augmented with Time-since-last-service (TSLS) counters. This algorithm not only promotes service regularity but also improves the original algorithm's delay performance.

The rest of this paper is organized as follows. We present related work and a summary of our contribution in Sections I-A and I-B, respectively. We describe the system model in Section II. We introduce and analyse the binary queue length based scheduling algorithms in Section III. In Section IV, we study another class of algorithms guaranteeing service regularity. We present the numerical results in Section V. We conclude with a mention of future directions in Section VI.

A. Related Work

Eryilmaz et al [2] generalized Max-Q to allow link weights to be certain functions of queue lengths or waiting times of the head-of-the-line (HoL) packets in the queues. They also showed that periodic or infrequent updates with *exact* queue length information (e.g., whenever the absolute difference with respect to the last reported value exceeds a threshold) suffice to achieve throughput optimality. Neely [3] combined queue length and HoL packet delay to ensure deterministic worst-case delay guarantees and to yield a throughput utility that differs from the optimal value by an amount that is inversely proportional to the delay guarantee. Ji et al [4] studied a HoL packet delay based back-pressure algorithm for multi-hop wireless networks. Ghaderi et al [5] proposed a MAC-layer queue length based maxweight scheduling algorithm addressing the cross layer interaction issue mentioned earlier. The optimality claims of queue length based algorithms hold under the assumption of a fixed network topology with each node having a stationary packet arrival process. Ven et al [6] showed that Max-Q suffers from *starvation* and *last-packet* problems and is not throughput optimal in presence of nonpersistent nodes. The HoL packet delay based algorithms do not have have this problem. Vargaftik et al [7] proposed a starvation free back-pressure algorithm that ensures repeated evacuation of all network queues. He et al [8] defined *age of information* at a node as the elapsed time since the most recently delivered update was generated and proposed an algorithm aimed at minimizing the overall information age.

Several recent works have studied throughput performance for flow level dynamics where fixed number of users with stationary packet arrivals are replaced by stationary arrival of finite-sized (or, short-lived) flows which leave the network after completing service. Sadiq and Veciana [9] took ages of files as weights and showed throughput optimality of the corresponding max-weight scheduling algorithm. Li et al [10] showed that the age based scheduling algorithm is optimal also when persistent and dynamic flows coexist. Chen et al [11] considered head-of-the-line access delay (which equals TSLS) of a flow as its weight but showed throughput optimality only under certain constraints on the transmission rates.

Li et al [12] used a linear combination of queue lengths and TSLS counters as link weights in order to design a scheduling algorithm aimed at reducing the variance of inter-service times. They showed that the proposed algorithm is throughput optimal and also guarantees service regularity. Recently, Li et al [13] extended this work for a flow level dynamics and established throughput optimality of an algorithm that uses only TSLS values to set link weights.

Link weights in Max-W are similar to ages of finite-sized flows (or, ages of HoL files of persistent flows) in [9], [10], [13]. However, note that ages in flow based models linearly grow before being irrelevant when the flows (or, files) complete service. On the other hand, a link weight in our model grows as long as the queue length is above a threshold, and reduces to 0 when the queue length is less than the threshold, i.e., the weights exhibit saw tooth patterns. We thus need more constraints on weight functions in our scheduling algorithm than in [9], [10]. Further, the notion of TSLS in our work differs from that in [12] in that we do not increment TSLS counters when the respective queue lengths are below a threshold,(it is the same threshold as mentioned above). The two notions coincide in the case of flow level models (e.g., [11], [13]). Due to this distinction, the proposed algorithm exhibits different regularity performance than [12]. Finally, the authors in [10] do not consider fading and in [13] only consider ON-OFF channel fading.

B. Our Contribution

1) We introduce a new class of throughput optimal max-weight scheduling algorithms which require only binary queue length information at the scheduler. For determining link weights we define new counters which exhibit quite abrupt dynamics. We introduce appropriate weight function to handle this.
2) We propose new notions of service regularity and time-since-last-service (TSLS)counter. We provide their relation. We also provide a lower bound on service regularity.
3) We introduce another class of throughput optimal max-weight scheduling algorithms that also guarantee service regularity. These algorithms use a combination of the above two counters to determine schedules.

II. Network Model

We consider a wireless network with L links where each link represents a transmitter receiver pair that are in communication range of each other. The network operates using a slotted time structure with slot boundaries indexed as $t = \{1, 2, 3, \dots\}$. Let random vector $A[t] \in \{0, 1, \dots, A_{\max}\}^L$ represent packet arrivals at slot t.[1] Let $C[t] \in \mathcal{C} \triangleq \{0, 1, \dots, C_{\max}\}^L$ denote channel state at slot t; $C_l[t]$ represents the number of packets that can be transmitted over link l at slot t. We capture fading by allowing $C[t]$ to be random. On the other hand, we model wireless interference using a link-based conflict model wherein a transmission over a link in a slot is successful if and only if its interfering links are not transmitting in the same slot. This model can be represented via a conflict graph whose vertices represent network links and each link is connected with all its interference links. We assume that packets arriving at a slot are available for service only from the next slot onwards. Let $S[t] \in \{0, 1\}^L$ denote the transmission schedule at slot t where $S_l[t] = 1$ if link l is scheduled in slot t and 0 otherwise. Let Ω be the collection of all possible schedules. Examples of our model include

(i) Cellular downlink (or uplink) with a common fading channel. Here only feasible schedules are singletons or the empty set.
(ii) Interference limited ad hoc networks. Here $C_l[t] = 1$ for all l, t.

We assume that both $A[t]$ and $C[t]$ are i.i.d. across slots with arrival rates $\mathbb{E}(A[t]) = \lambda$ and $\mathbb{P}[C[t] = c] = \pi_c$ for all $c \in \mathcal{C}$. Let $Q[t] \in \mathbb{Z}^L$ denote queue lengths at slot t and $Q[t-]$ denote queue lengths prior to adding the arrivals at slot $t-1$. These processes evolve as

$$\begin{aligned} Q_l[(t+1)-] &= (Q_l[t] - C_l[t]S_l[t])^+, \\ Q_l[t+1] &= (Q_l[t] - C_l[t]S_l[t])^+ + A_l[t]. \end{aligned} \tag{1}$$

We say that the network is stable if its state evolution is Markov chain,call the system stable if the underlying Markov Chain, is positive recurrent. In this it means $Q[t]$ being positive recurrent. The capacity region of a network is defined to be the set of arrival rates for which there exist scheduling algorithms that render the queues stable.

It is well known that our network has capacity region [1],

$$\Lambda \triangleq \{\lambda : \lambda_l = \sum_{c\in\mathcal{C}} \pi_c \sum_{S\in\Omega} \theta_{c,s} c_l S_l, \forall l = 1, 2, \dots L,$$
$$\text{for some } (\theta_{c,S})_{c\in\mathcal{C}, S\in\Omega} \text{ with } \sum_{S\in\Omega} \theta_{c,S} = 1, \forall c\} \tag{2}$$

Finally, we call a scheduling algorithm throughput optimal if it stabilizes the network queues for any arrival λ such that $(1+\epsilon)\lambda \in \Lambda$ for some $\epsilon > 0$.

A. Service Regularity

Service regularity is a notion of fairness that refers to variation of inter-service times at links. The scheduling algorithm should fairly treat the traffic injected into the MAC layer queues. With this perspective, inter-service time for a link

[1] We refer to the duration $[t, t+1)$ as slot t.

should exclude the slots at which the MAC layer queue at this link is below the threshold. So our notion of service regularity is different from [12] where inter-service time calculation does not take into account the queue states. We now formally define our notion of service regularity. Let $B_{l,m}, m = 1, 2, \dots,$ denote the slot at which mth service of link l begins. Let $\tau_{l,m}$ denote the earliest slot as seen at $B_{l,m+1}$ since when l's queue length has been greater than or equal to $C_{\max}$. More precisely, $\tau_{l,m} = \min\{t \leq B_{l,m+1} : Q_l[t-] \geq C_{\max}\}$. We then define

$$I_{l,m} \triangleq B_{l,m} - \max\{B_{l,m-1}, \tau_{l,m-1}\}$$

to be the mth inter-service time at l. Let us assume that the network is stable and, for each l, $I_{l,m}$ converge to a random variable $\bar{I}_l$. Motivated by the discussion in [12], we define a normalized second moment of $\bar{I}_l$, $\mathbb{E}[\bar{I}_l^2]/\mathbb{E}[\bar{I}_l]^2$, to be a measure of service regularity at link l.

In the following sections, we present a class of scheduling algorithms which ensure throughput optimality and service regularity. Throughout we assume that the scheduler knows instantaneous channel states.

III. Binary Queue Length Information based Scheduling

We propose a class of maxweight scheduling algorithms in which link weights, are updated only on the basis of queue lengths being greater or smaller than $C_{\max}$. More precisely, the scheduler maintains a counter $W_l(t)$ for each link l, which measures the time which l's queue length has been $C_{\max}$ or above. The scheduler uses $f(W_l(t)), l = 1, \dots, L$ as link weights - different algorithms in the proposed class are distinguished by the "weight functions" f that they use. The weight function is chosen from the following set:

$$\mathcal{F} = \Big\{ f : \mathbb{R}^+ \to \mathbb{R}^+ \text{ such that (i) } f(0) = 0, \\ \text{(ii) } f \text{ is concave, differentiable and increasing}, \\ \text{(iii) } \exists M < \infty \text{ such that } xf'(x) < M, \forall x. \Big\}$$

$\mathcal{F}$ includes $f(x) = (\log(1+x))^\theta$ for some $0 < \theta \leq 1, f(x) = \log\log(e + x)$ and $f(x) = \frac{\log(1+x)}{\log(e+\log(1+x))}$. Several earlier works have also proposed classes of weight functions for designing scheduling policies. The authors in [2], [10] allow a larger class of functions including $f(x) = x$. Note that $W_l(t)$ for a link l resets to 0 whenever $Q_l[t-]$ drops below $C_{\max}$, no matter how large its previous value was. The abrupt dynamics of these counters fundamentally differs from that of queue-lengths which renders many weight functions unusable in our context. In [13], the authors have prescribed the class $\mathcal{G}$ of functions $f : \mathbb{R}^+ \to \mathbb{R}^+$ with the following properties.

1) $f(0) = 0$,
2) f is concave, differential and increasing,
3) for any $k \geq 1, b \geq 0, \exists c < \infty$ such that $f(kx + b) \leq f(x) + c, \forall x$.

We make the following claim.

Lemma 3.1: $\mathcal{F} = \mathcal{G}$

Proof: See Appendix A. ∎

Having fixed an $f \in \mathcal{F}$, the algorithm works as follows at each slot.

Algorithm 1 Max-W

1. A link l informs the scheduler if $Q_l[t-]$ has reached $C_{\max}$ from below or $C_{\max} - 1$ from above.
2. The scheduler updates the counters $W_l(t)$ as
$$W_l(t+1) = (W_l[t] + 1)\mathbb{1}_{\{Q_l[(t+1)-] \geq C_{\max}\}}, \forall l \quad (3)$$
3. The scheduler chooses a $S^{(W)}[t]$ satisfying
$$S^{(W)}[t] \in \underset{S \in \Omega}{\operatorname{argmax}} \sum_{l=1}^{L} f(W_l[t]) C_l[t] S_l[t]$$

Remarks 3.1: 1) Let us note that MAC layer queues are typically larger than $C_{\max}$. Assume that the transport layer persistently sends packets to the MAC layer until total backlog is served. Then the MAC layer can learn whether the backlog is less than $C_{\max}$ by just observing it own queue. Note that $C_{\max}$ in (3) can be replaced with any threshold greater than $C_{\max}$.
2) Observe that the links do not need to communicate to the scheduler in every slot for $W(t)$ updates.
3) Also observe that a link l is served at slot t (i.e., $S_l[t] = 1$) only if $Q_l[t] \geq C_{\max}$. So queue-length evolution follows

$$Q_l[t+1] = Q_l[t] - C_l[t] S_l^{(W)}[t] + A_l[t]. \quad (4)$$

We next show that the proposed algorithms are throughput optimal.

Theorem 3.1: Max-W is throughput optimal, i.e., under Max-W, for all λ such that $(1 + \epsilon)\lambda \in \Lambda$ for some $\epsilon > 0$, the Markov chian $(Q[t], W[t])$ is positive recurrent. Also,

$$\limsup_{K \to \infty} \frac{1}{K} \sum_{t=1}^{K-1} \sum_{l=1}^{L} \mathbb{E}(f(Q_l[t])) < \infty.$$

Proof: Consider a Lyapunov function

$$V(W[t], Q[t]) = \sum_{l=1}^{L} f(W_l[t]) Q_l[t]$$

Note that

$$\begin{aligned}
&\sum_{l=1}^{L} f(W_l[t+1]) Q_l[t+1] \\
&\overset{(a)}{=} \sum_{l=1}^{L} f(W_l[t+1])(Q_l[t] - S_l^{(W)}[t] C_l[t] + A_l[t]) \\
&\overset{(b)}{\leq} \sum_{l=1}^{L} f(W_l[t] + 1)(Q_l[t] - S_l^{(W)}[t] C_l[t] + A_l[t]) \\
&\overset{(c)}{\leq} \sum_{l=1}^{L} (f(W_l[t]) + f^{'}(W_l[t]))(Q_l[t] - S_l^{(W)}[t] C_l[t] \\
&\quad + A_l[t]),
\end{aligned}$$

where $(a),(b)$ follow from (4) and (3) respectively, and (c) follows from concavity of f. The drift of the Lyapunov function

$$\begin{aligned}
&\Delta V[t] \\
&\triangleq V(W[t+1],Q[t+1]) - V(W[t],Q[t]) \\
&= \sum_{l=1}^{L} f(W_l[t+1])Q_l[t+1] - \sum_{l=1}^{L} f(W_l[t])Q_l[t] \\
&\leq \sum_{l=1}^{l=L} f(W_l[t])(A_l[t] - S_l^{(W)}[t]C_l[t]) + \sum_{l=1}^{l=L} f^{'}(W_l[t])Q_l[t] \\
&\quad + \sum_{l=1}^{l=L} f^{'}(W_l[t])(A_l[t] - S_l^{(W)}[t]C_l[t]) \\
&\overset{(d)}{\leq} \sum_{l=1}^{l=L} f(W_l[t])(A_l[t] - S_l^{(W)}[t]C_l[t]) \\
&\quad + \sum_{l=1}^{l=L} f^{'}(W_l[t])A_{\max}(W_l[t] + C_{\max}) + \sum_{l=1}^{l=L} f^{'}(W_l[t])A_{\max} \\
&\overset{(e)}{\leq} \sum_{l=1}^{l=L} f(W_l[t])(A_l[t] - S_l^{(W)}[t]C_l[t]) \\
&\quad + Lf^{'}(0)A_{\max}(C_{\max}+1) + \sum_{l=1}^{l=L} A_{\max} f^{'}(W_l[t])W_l[t] \\
&\leq \sum_{l=1}^{l=L} f(W_l[t])(A_l[t] - S_l^{(W)}[t]C_l[t]) + B, \qquad (5)
\end{aligned}$$

where $B = Lf^{'}(0)A_{\max}(C_{\max}+1) + A_{\max}ML$. Here (d) follows from $Q_l[t] \leq A_{\max}(W_l[t] + C_{\max})$ and $A_l[t] - S_l^{(W)}[t]C_l[t] \leq A_{\max}$, (e) follows from $f^{'}(W_l[t]) \leq f^{'}(0)$, and (e) follows from $xf^{'}(x) \leq M$. So, the conditional drift,

$$\begin{aligned}
&\mathbb{E}(\Delta V[t]|W[t],Q[t]) \\
&\leq \sum_{l=1}^{L} \mathbb{E}(f(W_l[t])(A_l[t] - S_l^{(W)}[t]C_l[t])|W[t]) + B \\
&= \sum_{l=1}^{L} \lambda_l f(W_l[t]) - \sum_{l=1}^{L} \mathbb{E}(f(W_l[t])(S_l^{(W)}[t]C_l[t])|W[t]) + B. \qquad (6)
\end{aligned}$$

For any λ, such that $(1+\epsilon)\lambda \in \Lambda$ for some $\epsilon > 0$,

$$\lambda_l \leq \sum_c \pi_c \sum_S \theta(c,S)c_l S_l - \epsilon, \forall l. \qquad (7)$$

Hence, we have

$$\begin{aligned}
&\sum_{l=1}^{L}(\lambda_l + \epsilon)f(W_l[t]) \\
&\leq \sum_c \pi_c \sum_S \theta(c,S) \sum_{l=1}^{L} f(W_l[t])c_l S_l \\
&\leq \sum_c \pi_c \sum_S \theta(c,S) \sum_{l=1}^{L} f(W_l[t])c_l[t] S_l^{(W)}[t] \\
&= \mathbb{E}\left[\sum_{l=1}^{L} f(W_l[t])C_l[t]S_l^{(W)}[t]|W_l[t]\right], \qquad (8)
\end{aligned}$$

where the second inequality follows from definition of Max-W. By substituting (8) in (6), we have

$$\mathbb{E}(\Delta V[t]|W[t],Q[t]) \leq -\epsilon \sum_{l=1}^{L} \lambda_l f(W_l[t]) + B.$$

By summing above inequality over $t = 1,2,\ldots K$ and taking expectation we have

$$\limsup_{K\to\infty} \frac{1}{K} \sum_{t=1}^{K} \sum_{l=1}^{L} \lambda_l \mathbb{E}[f(W_l[t])] \leq \frac{B}{\epsilon}. \qquad (9)$$

Since $Q_l[t] \leq A_{\max}(W_l[t] + C_{\max})$ and $f \in \mathcal{G}$ (see Lemma 3.1), there exists a $D > 0$ such that

$$\begin{aligned}
f(Q_l[t]) &\leq f(A_{\max}W_l[t] + A_{\max}C_{\max}) \\
&\leq f(W_l[t]) + D. \qquad (10)
\end{aligned}$$

So we can write,

$$\begin{aligned}
&\limsup_{K\to\infty} \frac{1}{K} \sum_{t=1}^{K} \sum_{l=1}^{L} \lambda_l \mathbb{E}[f(Q_l[t])] \\
&\leq \limsup_{K\to\infty} \frac{1}{K} \sum_{t=1}^{K} \sum_{l=1}^{L} \lambda_l \mathbb{E}[f(W_l[t]) + D] \\
&\leq \frac{B}{\epsilon} + D \sum_{l=1}^{L} \lambda_l < \infty. \qquad (11)
\end{aligned}$$

Inequalities (9) and (11) imply that the Markov chain $(Q[t],W[t])$ is positive recurrent [14]. ■

IV. Scheduling for Service Regularity

Our approach in this section is motivated by [12]. As in [12], we introduce another set of counters, $T[t] \in (\mathbb{Z}^+)^L$, referred to as times since last service; $T_l[t]$ denotes the number of slots for which l's queue length has been greater than or equal to the threshold,$C_{\max}$ and not served. Formally, it evolves as

$$T_l[t+1] = (T_l[t]+1)\mathbb{1}_{\{C_l[t]S_l[t]=0\}}\mathbb{1}_{\{Q_l[(t+1)-]\geq C_{\max}\}}. \quad (12)$$

Though our notions of service regularity and TSLS are different from [12], these two also share same relation as in [12, Lemma 1]. We state it below. Its proof is exactly same as in [12].

Lemma 4.1: If the sequences $(I_{l,m}, 1 \leq l \leq L)$ and $T[t]$ converge to random vectors $\bar{I}$ and $\bar{T}$, respectively, then

$$\mathbb{E}[\bar{T}_l] = \frac{1}{2}\left(\frac{\mathbb{E}[\bar{I}_l^2] - E[\bar{I}_l]}{\mathbb{E}[\bar{I}_l]}\right). \qquad (13)$$

In the following, we use an affine function of $\mathbb{E}[\bar{T}_l]$ as a measure of service regularity.

We first illustrate delay and regularity performances of Max-Q and Max-W algorithms via simulation. We consider a cellular downlink with 8 links. We consider Bernoulli arrivals with $\lambda_l = \frac{1}{(1.9)(1+l)}, l = 1,\ldots,8$ and assume no channel fading. We show links' delays and service regularity in Figure IV. We see that Max-W yields much better delays for links with smaller arrival rates and also provides much equitable regularity performance.

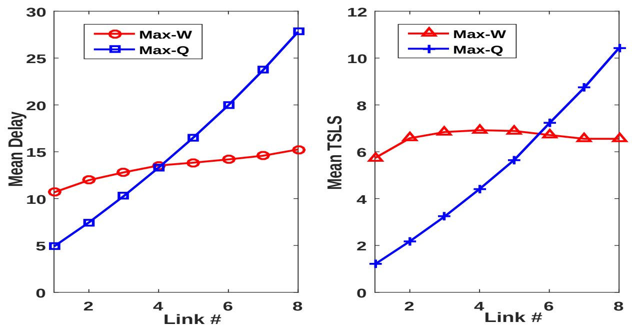

Fig. 1. Delay and Regularity performance of Max-Q and Max-W.

However, we find that regularity performance of all the links can be further improved (see the service regularity under the Max-WT in Figure 6). We should thus incorporate TSLS information in the scheduling decisions. The TSLS counter has direct impact on service regularity; smaller TSLS value imply more regular service.

We next show via an example that a maxweight scheduling algorithm using only TSLS as weights, i.e., choosing schedules as

$$S[t] \in \operatorname*{argmax}_{S\in\Omega} f(T_l[t])C_l[t]S_l[t], \tag{14}$$

is not throughput optimal. Let us consider a cellular downlink with 2 users (or, links). Let $C_1[t]$ and $C_2[t]$ be independent and identically distributed (in addition to being i.i.d across slots) Bernoulli random variables with $\pi_1 = \pi_0 = 0.5$. The capacity region of this network is

$$\left\{\lambda \in (\mathbb{R}^+)^2 : \lambda_1 < 0.5, \lambda_2 < 0.5 \text{ and } \lambda_1 + \lambda_2 < 0.75\right\}$$

Now consider an arrival rate λ such that λ_1 and λ_2 are very close to (but smaller than) 0.5 and 0.25, respectively. Such a λ is clearly within the capacity region. Also, in view of $\mathbb{E}(C_l[t]) = 0.5$, for stability, the first link should be served at almost every slot when $C_1[t] = 1$. However, $T_1[t+1]$ is reset to 0 after every t when link 1 is served. Hence, if $C[t] = (1, 1)$ but $Q_2[t+1] > 0$, the TSLS based algorithm will not schedule link 1. Since probability of this event is strictly positive, the above algorithm clearly cannot drive the network to stability.

A. Scheduling Algorithm Guaranteeing Regular Service

From the above discussion we find that we need to carefully balance the weights of W and T parameters to come up with algorithms that are throughput optimal and also give good service regularity performance. We propose the another class of scheduling algorithms parameterized by $f \in \mathcal{F}$ and control parameters $\alpha_l > 0, \beta_l \geq 0, l = 1, \ldots, L$ and $\gamma \geq 0$.

The algorithm works as follows at each slot.

Notice that, for $T[t]$ updates, the links need to inform the scheduler when their queue lengths cross the threshold, i.e, $C_{\max}$ from above or below. The parameters α_l and

Algorithm 2 Max-WT

1. A link l informs the scheduler if $Q_l[t-]$ has reached $C_{\max}$ from below or $C_{\max} - 1$ from above.
2. The scheduler updates $W_l(t)$ and $T_l(t)$ as in (3) and (12), respectively.
3. The scheduler chooses a $S^*[t]$ satisfying

$$S^*[t] \in \operatorname*{argmax}_{S\in\Omega} \sum_{l=1}^{L} (\alpha_l f(W_l[t]) + \gamma\beta_l T_l[t])C_l[t]S_l[t].$$

$\beta_l, l = 1, \ldots, L$, in Max-WT can be tuned to control various performance metrics, e.g., delay and service regularity, of different links. Setting $\gamma = 0$ reduces Max-WT to Max-W.

Theorem 4.1: Max-WT is throughput optimal, i.e., under Max-WT, Markov chain $(Q[t], W[t], T[t])$ is positive recurrent for all arrival rates λ such that $(1+\epsilon)\lambda \in \Lambda$ for some $\epsilon > 0$. Also,

$$\limsup_{K\to\infty} \frac{1}{K} \sum_{t=0}^{K-1} \sum_{l=1}^{L} \alpha_l \mathbb{E}(f(W_l[t])) \leq \frac{B(\alpha, \beta, \gamma)}{\epsilon}.$$

where $B(\alpha, \beta, \gamma) = \sum_{l=1}^{L} \alpha_l (M A_{\max} + f'(0) A_{\max}(C_{\max} + 1)) + \gamma C_{\max} \sum_{l=1}^{L} \beta_l$.

Proof: See Appendix B. ■

B. Service Regularity of Max-WT

We first state a lower bound on service regularity performance of any algorithm that renders the Markov chain $(Q[t], W[t], T[t])$ stable. Let $(\bar{Q}, \bar{W}, \bar{T})$ be random vectors having same distribution as $(Q[t], W[t], T[t])$ in steady state. We assume that second momemnt of $\bar{T}$ is bounded. Let us also introduce a random vector $\bar{C}$ having same distribution as $C[t]$ and define a random set $\bar{H} \triangleq \{l : \bar{C}_l \bar{S}_l > 0 \text{ or } \bar{Q}_l = 0\}$. The following lower bound holds for all the candidate algorithms. This bound is similar to [12, Proposition 1], and is obtained following similar arguments.

$$\sum_{l=1}^{L} \beta_l \lambda_l \mathbb{E}(\bar{T}_l) \geq \frac{1}{2} \left(\frac{\sum_{l=1}^{L} \beta_l \lambda_l}{\mathbb{E} \sum_{l\in\bar{H}} \beta_l \lambda_l} - 1 \right)$$

We now provide an upper bound on the service regularity under Max-WT. Here, $(\bar{Q}, \bar{W}, \bar{T})$ represent steady state random vectors under Max-WT.

Theorem 4.2: Under Max-WT, for any λ such that $(1+\epsilon)\lambda \in \Omega$ for some $\epsilon > 0$, we have

$$\sum_{l=1}^{L} \beta_l \lambda_l \mathbb{E}(\bar{T}_l) \leq \frac{C_{\max}}{1+\epsilon} \left(\sum_{l=1}^{L} \beta_l - \mathbb{E}\left[\sum_{l\in\bar{H}} \beta_l \right] \right) + \frac{1}{\gamma(1+\epsilon)} B \tag{15}$$

where $B = Lf'(0)A_{\max}(C_{\max} + 1) + A_{\max} ML$.

Proof: We omit the proof for brevity. It is along the lines of the proof of [12, Proposition 4] though we use a different Lyapunov function. ■

Note that, as γ goes to infinity, the upper bound reduces to the first term in the right hand side of (15). For an illustration, let us consider a cellular downlink with L links. Let $\beta_l = 1$ and $\lambda_l = \frac{1}{L(1+\epsilon)}$ for each link l. Then, for $\gamma \to \infty$, the upper bound on the service regularity under Max-WT is atmost $2\mathbb{E}(|\bar{H}|)$ times the lower bound, where $|\bar{H}|$ is cardinality of $\bar{H}$.

V. Simulation Results

In this section, we provide simulation results for Max-Q, Max-W and Max-WT algorithms and also discuss their performance. We use $f(x) = \log(1+x)$ throughout. We consider the following two setups.

1) Cellular downlink with 4 nodes and a ON-OFF fading channel. We assume that $C_l[t]$ equals $c > 0$ with probability 0.8 and 0 with probability 0.2. The capacity region of this network include

$$\left\{\lambda : \lambda_1 = \cdots = \lambda_4 = \rho\left(\frac{1-(1-p)^4}{4}\right), 0 \leq \rho < 1\right\}.$$

2) 4×4 grid network with 16 nodes and 24 links. We assume no channel fading and one-hop interference constraints. We take four maximal schedules represented by $S^1, S^2, S^3, S^4 \in \{0,1\}^{24}$. We then consider following arrival rates which clearly lie in the capacity region (see [15] for an elaborate description of this setup).

$$\left\{\lambda : \lambda = \frac{\rho}{4}\sum_{i=1}^{4} S^i, 0 \leq \rho < 1\right\}.$$

ρ in the above sets is referred to as load intensity. In each case, as $\rho \to 1$, the arrival rates approach a point on the boundary of the capacity region.

We first demonstrate throughput optimality of the algorithms. For this we consider the cellular downlink setup with $c = 3$ and $\rho = 0.9$. In Figure 2, we show aggregate queue length evolution for Max-Q, Max-W and Max-WT with $\gamma = 1, 5$. We see that the queues stabilize in each case.

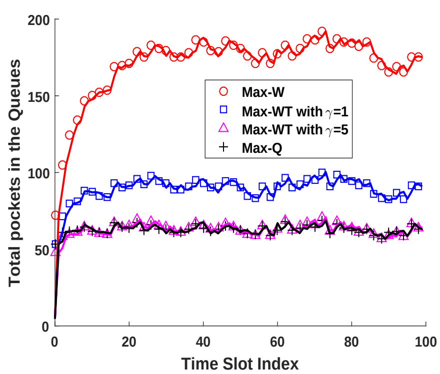

Fig. 2. Aggregate queue length evolution for the cellular downlink setup

We show mean queue lengths as a function of load intensity for the cellular and grid network setups in Figures 3 and 4, respectively. We have again set $c = 3$ in the cellular setup. We observe that Max-WT can also yield better delay performance than Max-W if the control parameter γ is appropriately selected.

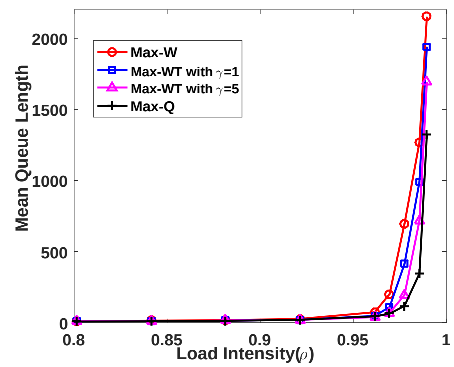

Fig. 3. Mean queue length vs load intensity for the cellular downlink setup

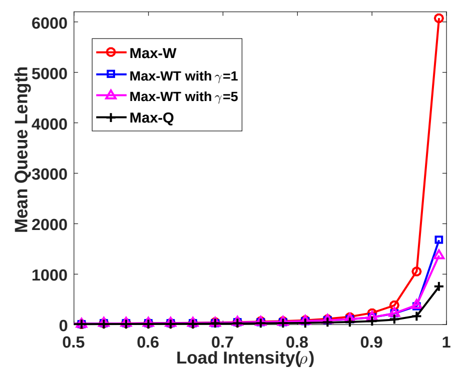

Fig. 4. Mean queue length vs load intensity for the grid network setup

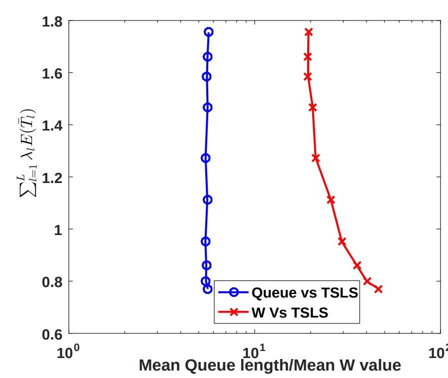

Fig. 5. Mean W - service regularity tradeoff; also shown is mean queue length- service regularity tradeoff

We show tradeoff between mean aggregate queue lengths and service regularity and between $\mathbb{E}(\sum W_l[t])$ and service regularity (see Figure 5). Here, we have considered cellular downlink with $c = 1$ and $\rho = 0.93$. Different points in the plots correspond to different γs which vary as $2^{-i}, i = -4, -3, \ldots, 4$. Expectedly, service regularity improves and $\mathbb{E}(\sum W_l[t])$ increases as γ is increased. We also observe that service regularity is not improving at the expense of mean queue lengths. Finally, we compare delay and service regularity performances of Max-W and Max-WT for the same setup used in Figure IV. We clearly see that Max-WT yields much better service regularity.

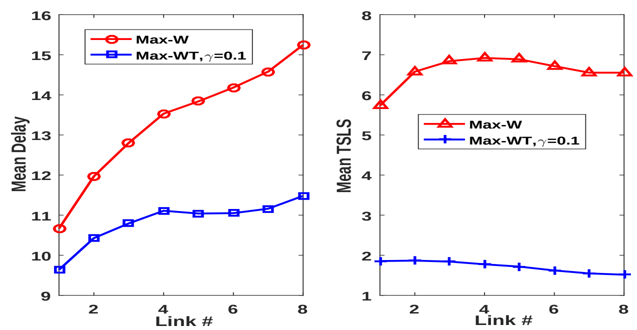

Fig. 6. Delay and Regularity performance of Max-Q and Max-W.

VI. Conclusion

We have proposed two classes of maxweight scheduling algorithms, Max-W and Max-WT, both using binary queue length information. Algorithms in both the classes are throughput optimal (Theorems 3.1 and 4.1) and Max-WT algorithms also guarantee service regularity (Theorem 4.2).

We would like to further study the regularity performance of Max-WT algorithms. We would also like to investigate delay performance of Max-W and Max-WT algorithms. Our future work also includes designing CSMA-type distributed versions of the proposed algorithms (similar to QCSMA for queue length based maxweight [16]). The CSMA algorithms assume smooth dynamics of the underlying weights (see [15]). The weights in our algorithms change abruptly. We would like to explore alternate dynamics suitable for distributed implementation. .

Appendix A
Proof of Lemma 3.1

We first show that $\mathcal{G} \subseteq \mathcal{F}$ and then that $\mathcal{F} \subseteq \mathcal{G}$. These together prove the claim.

$\mathcal{G} \subseteq \mathcal{F}$: Consider a $f \in \mathcal{G}$. Invoking the properties of $\mathcal{G}$, for $k = 2, b = 0, \exists c < \infty$ such that

$$f(2y) \leq f(y) + c, \forall y \geq 0. \tag{16}$$

Hence, for any $x \geq 0$,

$$\begin{aligned} xf(x) &= 2\left(\frac{x}{2} f'(x)\right) \\ &\leq 2\left(f(x) - f\left(\frac{x}{2}\right)\right) \\ &\leq 2c, \end{aligned}$$

where the first inequality follows because f is concave and increasing, and the second follows from setting $y = x/2$ in (16). Hence $f \in \mathcal{F}$. Since these arguments hold for any $f \in \mathcal{F}$, $\mathcal{G} \subseteq \mathcal{F}$.

$\mathcal{F} \subseteq \mathcal{G}$: Now consider a $f \in \mathcal{F}$. For any $k \geq 1, b \geq 0$ and for all x,

$$\begin{aligned} f(kx+b) - f(x) &\leq f'(x)((k-1)x + b) \\ &\leq (k-1)M + f'(0)b, \end{aligned}$$

where the first inequality follows because f is concave and increasing, and the second because $f'(x) \leq f'(0)$ and $xf'(x) \leq M$. We observe that any f in $\mathcal{F}$ is also in $\mathcal{G}$, and so $\mathcal{F} \subseteq \mathcal{G}$.

Appendix B
Proof of Theorem 4.1

The proof is along the lines of proof of [12, Proposition 1]. For improved readability we define $Z[t] \triangleq (Q[t], W[t], T[t])$, the Markov chain under consideration. Consider the Lyapunov function

$$V(Z[t]) \triangleq \sum_{l=1}^{L} \alpha_l f(W_l[t]) Q_l[t] + \gamma C_{\max} \sum_{l=1}^{L} \beta_l T_l[t],$$

and define $\Delta V[t] \triangleq V(Z[t+1]) - V(Z[t])$, Then, the conditional expected drift

$$\begin{aligned} &\mathbb{E}(\Delta V[t]|Z[t]) \\ &= \mathbb{E}(V(Z[t+1]) - V(Z[t])|Z[t]) \\ &= \mathbb{E}\left[\sum_{l=1}^{L} \alpha_l (f(W_l[t+1]) Q_l[t+1] - f(W_l[t]) Q_l[t])|Z[t]\right] \\ &\quad + \gamma C_{\max} \mathbb{E}\left[\sum_{l=1}^{L} \beta_l (T_l[t+1] - T_l[t])|Z[t]\right] \end{aligned} \tag{17}$$

Let $H^*[t] \triangleq \{l : C_l[t] S_l^*[t] > 0 \text{ or } Q_l[t] = 0\}$. Then

$$\begin{aligned} \sum_{l=1}^{L} \beta_l T_l[t+1] &= \sum_{l \notin H^*[t]} \beta_l (T_l[t] + 1) \\ &\leq \sum_{l=1}^{L} \beta_l T_l[t] + \sum_{l=1}^{L} \beta_l - \sum_{l \in H^*[t]} \beta_l T_l[t] \end{aligned} \tag{18}$$

By substituting (18) in (17), and following similar steps that we used to obtain (5),

$$\begin{aligned} &\mathbb{E}(\Delta V[t]|Z[t]) \\ &\leq \sum_{l=1}^{L} \alpha_l \mathbb{E}(f(W_l[t])(A_l[t] - S_l^*[t] C_l[t])|Z[t]) \\ &\quad - \gamma C_{\max} \sum_{l \in H^*} \mathbb{E}(\beta_l T_l[t]|Z[t]) \\ &\quad + \gamma C_{\max} \sum_{l=1}^{L} \beta_l + \sum_{l=1}^{L} \alpha_l (M A_{\max} + f^{'}(0) A_{\max}(C_{\max} + 1)) \\ &= \sum_{l=1}^{L} \alpha_l \lambda_l f(W_l[t]) - \mathbb{E}(\sum_{l=1}^{L} \alpha_l f(W_l[t] C_l[t] S_l^*[t]|Z[t]) \end{aligned}$$

$$- \gamma C_{\max} \sum_{l \in H^*} \mathbb{E}(\beta_l T_l[t] | Z[t]) + B(\alpha, \beta, \gamma), \tag{19}$$

where $B(\alpha, \beta, \gamma)$ is as in the statement of the Theorem 4.1. Let $S^{(W)} \in \text{argmax}_{S \in \Omega} \sum_{l=1}^{L} f(W_l[t]) C_l[t] S_l[t]$ be a schedule that Max-W would choose. Then, by the definition of Max-WT,

$$\sum_{l=1}^{L} (\alpha_l f(W_l[t]) + \gamma \beta_l T_l[t]) C_l[t] S_l^*[t]$$
$$\geq \sum_{l=1}^{L} (\alpha_l f(W_l[t]) + \gamma \beta_l T_l[t]) C_l[t] S_l^{(W)}[t].$$

So,

$$\sum_{l=1}^{L} \alpha_l f(W_l[t]) C_l[t] S_l^*[t] \geq \sum_{l=1}^{L} \alpha_l f(W_l[t]) C_l[t] S_l^{(W)}[t] - \gamma \beta_l T_l[t] C_l[t] S_l^*[t]. \tag{20}$$

By substituting (20) in (19),

$$\begin{aligned}
&\mathbb{E}(\Delta V | Z[t]) \\
&\leq \sum_{l=1}^{L} \alpha_l \lambda_l f(W_l[t] - \mathbb{E}\left[\sum_{l=1}^{L} \alpha_l f(W_l[t] C_l[t] S_l^{(W)}[t])\right] \\
&\quad + \gamma \mathbb{E}\left[\sum_{l=1}^{L} \beta_l T_l[t] C_l[t] S_l^*[t] | Z[t]\right] \\
&\quad - \gamma C_{\max} \mathbb{E}\left[\sum_{l \in H^*[t]} \beta_l T_l[t] | Z[t]\right] + B(\alpha, \beta, \gamma) \\
&\leq \sum_{l=1}^{L} \alpha_l \lambda_l f(W_l[t]) - \mathbb{E}\left[\sum_{l=1}^{L} \alpha_l f(W_l[t] C_l[t] S_l^{(W)}[t])\right] \\
&\quad + B(\alpha, \beta, \gamma),
\end{aligned}$$

where the last inequality holds because we have

$$C_{\max}\left[\mathbb{E} \sum_{l \in H^*[t]} \beta_l T_l[t] | Z[t]\right] \geq \mathbb{E}\left[\sum_{l=1}^{L} \beta_l T_l[t] C_l[t] S_l^*[t] | Z[t]\right].$$

Now, for any λ such that $(1+\epsilon)\lambda \in \Lambda$ for some $\epsilon > 0$, arguing as in the proof of Theorem 3.1, we can show

$$\limsup_{K \to \infty} \frac{1}{K} \sum_{t=1}^{K} \sum_{l=1}^{L} \alpha_l \lambda_l \mathbb{E}[f(W_l[t])] \leq \frac{B(\alpha, \beta, \gamma)}{\epsilon}. \tag{21}$$

Similar to Max-W, we have $Q_l[t] \leq A_{\max}(W_l[t] + C_{\max})$, so following the arguements that led to (11), we have

$$\limsup_{K \to \infty} \frac{1}{K} \sum_{t=1}^{K} \sum_{l=1}^{L} \alpha_l \lambda_l \mathbb{E}[f(Q_l[t])] \leq \frac{B(\alpha, \beta, \gamma)}{\epsilon} + D \sum_{l=1}^{L} \alpha_l \lambda_l < \infty \tag{22}$$

Finally, since $T_l[t] \leq W_l[t]$, we can also claim

$$\limsup_{K \to \infty} \frac{1}{K} \sum_{t=1}^{K} \sum_{l=1}^{L} \alpha_l \lambda_l \mathbb{E}[f(T_l[t])] \leq \frac{B(\alpha, \beta, \gamma)}{\epsilon}. \tag{23}$$

Inequalities (21), (22) and (23) imply that the Markov chain $(Q[t], W[t], T[t])$ is positive recurrent [14].

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